J. Fluid Mech. (1975), *vol.* 71, *part* 4, *pp.* 815–826
Printed in Great Britain

Three-dimensional turbulent boundary layers in external flows: a report on Euromech 60

By T. K. FANNELØP AND P. Å. KROGSTAD
Division of Aero- and Gas-Dynamics, The Technical University of Norway, Trondheim

(Received 30 June 1975)

Euromech 60 is the third in a series of European Mechanics Colloquia dealing primarily with three-dimensional turbulent boundary layers. The Colloquium was held in Trondheim at the Technical University (NTH) from 14–16 April 1975 with forty-two participants from ten different countries, and was organized by L. N. Persen and T. K. Fanneløp. A total of 23 papers were presented, dealing with both experiments on and predictions of turbulent boundary-layer flows and related topics. Those concerned with the development of prediction methods were challenged by a set of boundary-layer flow problems defined well in advance in order to be calculated prior to the Colloquium. The results show close agreement for some calculated variables and surprisingly large discrepancies in others. A brief account of this exercise is included and it will also be the subject of a special report.

1. Introduction

Three-dimensional boundary layers were the main topic for Euromech 2 (Liverpool, 1966) as well as for Euromech 33 (Berlin, 1972). Whereas the colloquium at Liverpool dealt with both laminar and turbulent flows on stationary and rotating surfaces including flows in corners and ducts, at the Berlin meeting the field of interest had narrowed to include primarily only turbulent flows on stationary surfaces. The scope of Euromech 60 was further reduced in that only turbulent flows on external surfaces were included to avoid possible conflict with other Euromech Colloquia scheduled or planned on 'Boundary Layers in Turbomachines' and 'Internal Flow in Ducts, Pipes and Diffusers'.

The three-day programme was divided into six separate sessions. One full session was devoted to the predictions received for the test cases defined prior to the Colloquium. This exercise, informally known as the 'Trondheim Trials', will be discussed in a special report to be published in collaboration with the editors of the Eurovisc Annual Report 1975. No further publication of the proceedings is planned by the organizers.

2. Turbulence models

Rational efforts towards the prediction of turbulent boundary-layer flows include the examination of hypotheses and data to effect closure of the governing equations as well as the study of methods by which the solution to these equations can be found. The turbulence models put forth for three-dimensional turbulent flows are usually generalizations and extensions of two-dimensional models. This is justified, at least from an engineering point of view, by the fact that turbulence is a three-dimensional phenomenon even for nominally two-dimensional flows and complete data sets for genuinely three-dimensional boundary-layer flows are scarce if not completely non-existent.

J. P. Bernard proposed a four-equation model where the quantities k, ϵ, $\overline{uv}$ and $\overline{vw}$ are determined from transport equations. The symbols represent respectively the turbulent kinetic energy, the energy dissipation rate and the dominant Reynolds stresses in a three-dimensional boundary layer. The formulation is unusual in the sense that other well-known and accepted relations are recovered in the limits $y \rightarrow \delta$ and $y \rightarrow 0$. In the former the entrainment relations of Head (suitably modified to account for three-dimensional effects; see Smith 1974) are obtained whereas the inner limit yields the wall equilibrium conditions. The outer limit is based on the hypothesis that $(1-u)/(\partial u/\partial \eta) \rightarrow 0$ and $w/(\partial w/\partial \eta) \rightarrow 0$ as $y \rightarrow \delta$, which, it is claimed, has been verified by experiments.

W. Rodi proposed a much simpler model. To this end he presented the results of a long series of calculations aimed at verifying the validity of a high-Reynolds-number version of the so-called k, ϵ model (Launder & Spalding 1972). The model has great generality and can be considered intermediate between eddy-viscosity and transport-equation models. It relies on the Boussinesq assumption and the variable ϵ is considered 'isotropic'. This two-equation model contains five empirical constants, which have been evaluated by computer optimization using known experimental results for comparison with predicted quantities. The model has been tested successfully against all the two-dimensional flow problems of the Stanford Conference (1968) and certain two- and three-dimensional flows in ducts and cavities. Extensive calculations on three-dimensional turbulent boundary-layer flows were also presented as part of the Trondheim Trials.

New experimental information on three-dimensional turbulent boundary-layer flows is not often forthcoming, at least not in terms of quantities of direct interest to those concerned with the development of prediction methods. The measurements of six Reynolds stresses in incompressible turbulent boundary-layer flows presented by A. Elsenaar provided new information on problems and hypotheses discussed at Euromech 33. East presented on that occasion experimental evidence of the fact that the eddy viscosity could not be considered isotropic. The component in the direction normal to the local mean velocity appeared to be smaller by approximately 60 % than that in the direction of the mean velocity vector.

The new measurements indicate that the so-called East factor is not a constant, but increases as separation is approached. Experimentally the quantity is well defined only in the central region of the boundary layer. The shear-stress level is found to be lower for three-dimensional than for two-dimensional flows, while the mixing length appears to be reduced with increasing cross-flow. Turbulence models based on eddy-viscosity or mixing-length concepts are widely used in prediction methods for turbulent flows, and Elsenaar's data provide a new test case for the validity and accuracy of three-dimensional extensions of such models. The measurements were obtained with hot-wire probes from the same experimental swept-wing rig as was used by van den Berg & Elsenaar (1972). The new results complement the mean flow velocity measurements previously made, providing turbulence intensities and shear-stress profiles.

3. Computational methods

The trend in the development of prediction methods for three-dimensional turbulent flows favours the differential as opposed to the integral approach. Impressive applications of new and existing integral methods were presented at the Colloquium, but most of the new methods were based on finite-difference techniques and designed to exploit the capability of the modern digital computer.

Of the many papers concerned with differential methods, three represent studies and viewpoints of rather general character. In the paper which opened the Colloquium D. B. Spalding presented a very useful scheme for classification of three-dimensional viscous flows of boundary-layer and boundary-region type. This classification also leads to certain strategies of computation based on marching methods, with either a single sweep or repeated iterations to account for pressure interaction effects. The three-dimensional viscous flows of interest are those for which diffusion occurs mainly in directions normal to the external flow vector. In Spalding's shorthand notation such flows can be classified as 2D, $2\frac{1}{2}$D, 3D and 3D partially parabolic flows. In arriving at this classification, Spalding examined the influence of convection, diffusion and pressure interaction. The 2D class embraces the usual two-dimensional boundary layers whereas his $2\frac{1}{2}$D flows are those customarily referred to as three-dimensional boundary layers. The latter category includes flows for which the thin-boundary-layer approximation is valid and diffusion occurs in only one direction. Such flows offer considerable simplification over fully three-dimensional flows. The class of 3D flows of particular interest to Spalding is that denoted 'partially parabolic', which, as the label implies, can be solved by marching methods. For such flows diffusion in the streamwise direction can be ignored, but the pressure is not considered constant across the layer or independent of the viscous flow. The basic ideas expressed in Spalding's lecture have been published previously (Patankar & Spalding 1972). At this Colloquium Spalding gave a detailed discussion of how the method can be applied to predict the viscous flow near a ship's stern. It is evident from new experimental information presented at the Colloquium (by L. Larsson) that this problem is beyond the reach of '$2\frac{1}{2}$D boundary-layer theory'. The ship's stern problem had not been completed, but Spalding did present results for the flow in a curved duct, where the $2\frac{1}{2}$D approach proved inaccurate. The 3D partially parabolic formulation on the other hand gave excellent results with reasonable economy in terms of computer time.

The mathematical nature of the three-dimensional boundary-layer equations was also of concern to D. Humphreys. The question raised concerned what penalties will have to be paid in terms of loss in accuracy and validity of the calculated solution if the hyperbolic nature of the governing equations is not fully accounted for in the sense that the region of calculation does not contain the full domain of dependence. This problem is of interest for certain methods based on streamline co-ordinates (including the approximation of small cross-flow) where only the upstream region adjacent to the inviscid streamline is included. Such methods produce in many cases results which are on a par with or better than those of fully three-dimensional methods when compared with experimental results, and Humphreys attempted to explain why the approximation seems to work.

E. Krause & W. Kordulla were particularly concerned with the numerical accuracy of many of the usual finite-difference approximations. Their calculations show that plausible numerical techniques now widely used can give rise to errors of magnitude comparable to the differences associated with different turbulence models. Krause stressed the need to include the full domain of dependence in the numerical calculations. The presentation was illustrated by results obtained from a study of swept-wing boundary layers using various turbulence models and numerical schemes of both second- and fourth-order accuracy.

G. Schneider presented a new finite-difference method for incompressible turbulent flows. Implicit (five-point) finite-difference quotients are employed and the momentum difference equations are linearized to remove the coupling to allow the solution to be obtained through successive iterations. Physical co-ordinates are used and the turbulence model is adapted from the mixing-length formulae of Michel, Cousteix & Quémard (1971). The method has been applied to some of the test cases proposed for the Trondheim Trials.

Another new finite-difference procedure was presented by J. Cousteix & R. Michel. In this method the approximation of small cross-flow is invoked to decouple the two momentum equations, and an isotropic eddy-viscosity model is used to calculate the turbulent stresses. The method is not general but rather tailored to the problem of boundary-layer flows about infinite swept wings. The main purpose in developing this method was to check the validity and accuracy of a general integral method for solving three-dimensional turbulent boundary-layer problems. This leads naturally to the topic of integral methods.

A definite weakness associated with the use of integral methods is the need to choose the relevant velocity profiles in advance from restricted classes or families. This problem is more serious in three-dimensional flows because the cross-flow velocity profiles show such wide variations both in shape and magnitude, and reversals in the direction of the cross-flow are not uncommon. To avoid an arbitrary and perhaps unfortunate choice of velocity profiles, Cousteix & Michel have developed methods by which the related similarity solutions to a given problem can be used as profiles in the integral method. On the basis of these profiles the closure relations required in the entrainment integral method can be defined. This integral method, which is not restricted to small cross-flows, is written in terms of general co-ordinates in the manner suggested by Myring (1970). More details may be found in the doctoral dissertation by Cousteix (1974). The power and applicability of this method was amply demonstrated in a companion paper by V. Schmitt & J. Cousteix. The problem considered is that of laminar, transitional and turbulent flows about a finite swept wing at high angles of attack. The investigation is primarily experimental and will be dealt with in a later section, but it is worth notice that the predictions in this rather difficult case are in reasonable agreement with the measured values.

The entrainment integral method developed by P. D. Smith is similar to and predates that of Michel's group. The velocity profiles are of the power-law type in the streamwise direction and Mager's or Johnston's profile families are used in the cross-flow direction. The Ludwieg–Tillmann law is used to calculate the shear stress at the wall. The different cross-flow profiles produce rather different results, but Smith presented rational arguments for which family to choose in a given case. The method is well documented, see Smith (1974), and has been applied to a number of cases including all the cases defined for the Trondheim Trials.

The last integral method discussed at the Colloquium is difficult to categorize. It represents an extension of an earlier method developed for two-dimensional compressible turbulent flows by White & Christoph (1971). The new version by F. M. White, G. H. Christoph & R. C. Lessmann is restricted to the prediction of skin friction and cross-flow angles. The streamwise velocity profile is assumed to follow the wall law even in the outer part of the boundary layer, and the cross-flow angle and shear stress are calculated from Mager's hodograph profiles expressed in wall variables. The integration of the two momentum equations in the direction normal to the wall reduces the problem to two coupled equations for the unknown shear-stress components. This unorthodox approach has been shown to give reasonable answers in many cases of interest, but no profile parameters can be obtained by this method.

4. Experimental investigations

Nine experimental investigations were described at this Euromech Colloquium; of these the measurements by V. Schmitt & J. Cousteix, by A. Elsenaar, by J. Kux & H. P. Hoffman and by L. Larsson had been completed just prior to Euromech 60. These investigations all give extensive information on different aspects of three-dimensional turbulent boundary layers.

L. F. East presented a progress report on a hot-film probe specifically designed for use in three-dimensional turbulent boundary layers. The probe consists of two quartz rods, 0·15 mm in diameter, mounted in a V in a plane parallel to the wall and on each rod is a pair of thin films. The films are operated independently and after calibration of the probe analog voltages corresponding to the three components of the turbulence can be determined simultaneously.

The Reynolds stresses are then obtained by correlation. Stress measurements made with the probe in a two-dimensional boundary layer have been compared with measurements made with an X-wire probe and reasonable agreement found over most of the layer. Measurements have also been made in two strongly perturbed three-dimensional flows. They show that the direction of the shear stress changes slowly as the flow develops and lags considerably behind the changing direction of the velocity gradient. While it is not possible to check the accuracy of the turbulence measurements of any probe in a three-dimensional boundary layer it is noted that the stress measurements of Elsenaar using hot wires and of East using the hot-film probe display similar characteristics.

Two experimental programmes on ship hulls were reported. The investigation proposed by L. Larsson at Euromech 33 had now been concluded, resulting in a doctoral thesis. The measurements were performed in air on a reflex model of a cargo ship with a single hot wire. (A reflex model consists of the underwater part of the hull and its reflexion in the waterline.) Each measuring station was traversed twice with the wire axis in two different directions, thus giving the local flow direction. The skin friction was estimated from Clauser plots and compared with a number of different prediction methods of the integral type. For most of the methods checked, Larsson found good agreement with measurements. Detailed information about the measurements and a thorough survey of prediction methods for boundary layers on ship hulls can be found in his doctoral thesis (Larsson 1974).

A similar series of measurements has been undertaken by J. Kux & H. P. Hoffmann. At about 250 hull stations they recorded the static pressure and completed boundary-layer traverses with four different pressure probes. The static and total pressure distribution across the layer were measured and the mean flow velocity deduced from these measurements. The mean flow direction was further checked with a three-hole and a five-hole probe. Both Larsson and Kux & Hoffman compared their measured pressure distribution on the hull with the results of potential-flow calculations based on the method of Hess & Smith (1967). The agreement was generally very good.

The objective of the experiment initiated by P. Å. Krogstad is to add another case to the meagre library of test cases that can easily be used for checking three-dimensional turbulent boundary-layer calculation methods. Configurations of simple geometries are placed normal to a flat plate, thus generating three-dimensional viscous flow fields on the plate including regions of separated flow. The geometries of the models are chosen such that the potential flow is easily omputed. Blunt, cusped and wedge-shaped bodies are considered. Only flow visualization and theoretical flow-field studies were presented at the Colloquium.

V. Schmitt & J. Cousteix presented a detailed study of the flow over a swept tapered wing up to high angles of attack. The investigation was divided into three parts. The first included a detailed comparison between measured and computed potential-flow variables. The measured pressure distribution showed excellent agreement with predictions and the predicted and measured streamline directions showed noticeable deviations only near the tip of the wing. The second part consisted of a study of certain viscous phenomena such as boundary-layer transition, separation bubbles, the position of the leading-edge attachment line, wall streamlines, etc. These studies were performed by means of flow visualization at angles up to 15° and the information obtained was used in subsequent predictions. The third part of the study was devoted to the measurements of mean flow velocity profiles by means of Pitot tubes and very good agreement was again obtained with the theoretical predictions based on the integral prediction method referred to in an earlier section.

A. Bertelrud presented detailed plans for a wind-tunnel experiment where the model is the actual tip section of a SAAB A-32 Lansen wing. The purpose of this study is to obtain information on the effect of the Reynolds number and surface roughness on transition, turbulent boundary-layer growth and separation on an aeroplane component which has deteriorated through years of service. Results were not available at the time of the Colloquium.

5. Special topics

Under this heading we include a number of investigations not directly concerned with three-dimensional turbulent boundary-layer flows, but of great interest in their understanding and prediction. The topics of interest are stability and transition to turbulence as well as the organized structure of certain turbulent flows and the rate of decay of three-dimensional turbulent stresses.

Few problems are of greater interest to an aerodynamicist than the prediction of the onset of turbulent flow on wings and streamlined bodies. A review of the present state of the art of understanding and predicting transition onset, with particular reference to swept wings, was given by E. H. Hirschel. He presented a rather pessimistic view and concluded that we do not have satisfactory knowledge about stability in three-dimensional boundary layers and we know even less about the transition process. Parametric investigations indicate the possibility that the cross-flow instability may play a dominant role in causing turbulence on swept wings. He then presented the various criteria suggested to date for three-dimensional transition; of these he considered Owen & Randall's 'cross-flow Reynolds number' criterion the most useful. It is perhaps characteristic of the state of affairs in 1975 in this important field that the most trustworthy criterion has been revived from an old unpublished Ministry of Aviation report.

In order to understand the general problem of three-dimensional stability and transition, it may be necessary to study quite simple configurations and cases. To this end E. Palm presented a study of the stability of certain linear inviscid flows to finite three-dimensional disturbances independent of the streamwise co-ordinate. By an elegant and physically appealing argument he showed that these disturbances lead to instabilities even in the absence of inflexion points in the velocity profile. The argument applies also to the case of channel or pipe flows and Palm suggested that the phenomenon described could be responsible for the breakdown of Görtler vortices into turbulence. The periodic disturbances associated with Görtler vortices and similar phenomena are often present when least expected. Such disturbances appear in many nominally two-dimensional configurations, but they have not in the past been associated with flow in circular pipes. Evidence of a periodic flow structure in turbulent pipe flows was presented, however, by H. Thomann. These regular disturbances appear to originate in the inlet section, but they proved very difficult to eliminate. Thomann suggested that a periodic structure of the type observed is present in most 'two-dimensional' experiments but that it can be observed only if the mean flow is very steady. Further details will appear in a forthcoming ETH doctoral dissertation by Pozzorini.

In contrast to the efforts referred to above, L. N. Persen has in the past been concerned with the generation and study of streamwise-directed vortices. In previous publications (1970, 1971) he has shown how the flow patterns or cross-hatchings associated with re-entry flight can be duplicated by a simple water analogy. Persen suggested that information important in the study of turbulent boundary-layer flows can be extracted from the streamwise-directed vortices. In particular he suggested that the co-ordinate systems for three-dimensional prediction methods be aligned with the direction of these vortices. The vortex paths as indicated by striations on the surface have proved to be very stable and repeatable.

With an appropriate change in boundary conditions, a three-dimensional turbulent flow will relax to a nominally two-dimensional flow. The decay rates for the turbulent stresses are of considerable interest, and G. de Grande has studied such rates for a particular configuration: a curved duct turning through an angle of 60° followed by a straight section. The experimental set-up is similar to that used by Vermeulen (1971). The instruments used are a Pitot tube (for mean flow data) and a rotatable hot wire. The turbulent stresses, mean velocity, skewing angle and several thicknesses have been measured. The data show a relatively rapid decay of the three-dimensional characteristics, e.g. the skewing angle is reduced from 30° to 5° over a distance of only 1200 mm. The profiles were found to agree well with those proposed by Johnston.

6. Trondheim Trials

The idea of a three-dimensional counterpart to the 1968 Stanford Conference on two-dimensional turbulent boundary layers has undoubtedly occurred to many, and a specific proposal to this effect was put forward in the final discussions of Euromech 33. In planning Euromech 60 it became apparent that at present one would have to be content with an exercise more modest in scope, in the number of test cases and in the number of participants. Prof. Fernholz, Prof. Michel, Dr East and Dr Rotta kindly agreed to serve with one of us (T.K.F.) on the committee which selected the test cases. Those finally selected were in certain respects deficient. For one group, denoted 'theoretical cases' (mainly in the yawed-wing category), the initial and boundary conditions could be specified with precision, but the correct physical answers were unknown and may even be difficult to establish from experiments. These cases were intended primarily to give a comparison between methods, but agreement would not necessarily mean that the mathematical models predict the correct physics. A second group included cases for which measurements existed but the initial and boundary conditions were not always fully specified. Only in one case (van den Berg & Elsenaar's 1972 yawed-wing experiment) could the experimental information be considered satisfactory, but in the course of the Trondheim Trials serious questions were raised with regard to the accuracy of the measured pressure distribution.

In spite of these shortcomings, the Trondheim Trials were considered a success by the participants and represent a realistic demonstration of the present state of the art in Europe. Of the nine methods entered, only three (two integral and one difference method) could do all seven problems in the time available. The solutions were due six weeks prior to the Colloquium in order to give the chosen editor time to compile, edit and present the results. A list of participants is given in table 1.

Integral methods			
P. D. Smith	R.A.E.	J. Cousteix R. Michel	ONERA/CERT
F. M. White	University of Rhode Island		
Differential methods			
G. Schneider	DFVLR/AVA	T. K. Fanneløp D. Humphreys	FFA and NTH
J. Cousteix R. Michel	ONERA/CERT	W. Kordulla E. Krause	RWTH
W. Rodi A. K. Rastogi	University of Karlsruhe		
A. K. Rastogi D. B. Spalding A. Singhal	Imperial College	B. van den Berg P. F. Lindhout	NLR

TABLE 1

Space does not permit a detailed discussion of the calculations; for this we refer to the forthcoming report by the editor, L. F. East. But it is worth notice that the calculated results differ considerably in some cases, and furthermore, while some variables show good agreement, notably θ, other parameters such as the shape factor show substantial differences for the very same case. The spread in calculations among the various integral and differential methods is also quite substantial and no definite conclusion regarding the two categories can be made. In the case of van den Berg & Elsenaar's (1972) yawed-wing experiment, only one integral method (P. D. Smith's) predicts separation with the prescribed (measured at wall) pressure distribution. With this method separation occurs too early with one cross-flow profile family (Johnston's) and not at all with another (Mager's). All other methods fail to predict separation. By accounting for the displacement-thickness interaction (slightly inconsistent since the experimental pressure data were used) B. van den Berg & P. F. Lindhout came quite close to the measured results. The procedure used gives a detailed variation in pressure near the separation line not obtainable from the measured values. Equally good results were obtained by T. K. Fanneløp & D. Humphreys using the measured pressure data external to the boundary layer and a reduced eddy viscosity in the cross-flow direction. J. Cousteix & R. Michel obtained substantial improvements by modifying their method to take account of the pressure rise across the boundary layer as separation is approached.

At this point it is difficult to ascertain which one, if any, of these modifications will lead to improved predictions in the general case. The only safe conclusion which can be drawn is that the predicted separation location is very sensitive both to the prescribed pressure distribution and to the assumptions made. Operational three-dimensional methods giving reliable predictions of turbulent boundary-layer separation in the general case are apparently not yet in existence.

7. Conclusions

In the final paper of the Colloquium H. Fernholz, as chairman of Euromech 33, had been invited to give his personal views on the progress made in the intervening years. He noted that the balance between experimental and theoretical efforts in this field had changed somewhat in favour of theoretical and computational work. While he regretted the decline in the number of experimental investigations, the development in computational methods gave cause for optimism. The investigations of finite swept tapered lifting wings reported by P. D. Smith and V. Schmitt & J. Cousteix were particularly impressive in Fernholz' view. With regard to improvements in closure relations, there had been little or no progress. But he hoped that the new data on Reynolds stresses of the type reported by A. Elsenaar and L. F. East could form the basis for new or improved turbulence models.

Fernholz felt that the numerical study reported by E. Krause & W. Kordulla was long overdue. The relative merits of different turbulence models and methods of solving the equations cannot be determined unless the numerical problems are well understood. More information is needed on topics such as the domain of dependence, convergence, accuracy, grid independence, etc. Judging from the contributions to the Colloquium, he noted that little or no work is at present being undertaken in Europe on boundary layers on rotating bodies, on corner flows or on similar complex three-dimensional shear flows. Fernholz thought this rather regrettable as there is yet much to learn from the study of such problems. Other problems in three-dimensional boundary-layer flows which require intensified efforts are, in his view, compressible turbulent flows, transition to turbulence and separation of turbulent flows. He made a strong plea for giving research workers in different laboratories and countries in Europe a chance to discuss any new experimental programme before it is started. In this way one can be sure that the experiment will answer well-posed questions concerning the structure of the turbulent flow in a three-dimensional boundary layer.

Fernholz' paper set the stage for the final discussion. Quite optimistic views were expressed with regard to the technical problems discussed if only the present momentum could be sustained. A strong interest was expressed in having another colloquium on the same topic in about three years time, but no decision was made in this regard.

The Norwegian Institute of Technology (NTH) supported the meeting in many ways and contributed in large measure to its success.

The authors have drawn freely on abstracts, papers and notes provided by the participants of the Colloquium, including a draft report by Dr East on the results of the Trondheim Trials.

The set of test cases is available on request.

REFERENCES

(An asterisk by a name indicates a paper presented at the Colloquium)

BERG, B. VAN DEN & ELSENAAR, A. 1972 Measurements in a three-dimensional turbulent boundary layer under swept-wing conditions. *N.L.R. Amst. Tech. Rep.* no. 72092.

BERNARD, J. P.* Modélisation de la couche limite tridimensionelle turbulente et le rôle de la fonction d'entrainement.

BERTELRUD, A.* Wind-tunnel measurements of the flow around the outer part of a swept wing.

COUSTEIX, J. 1974 Analyse théorique et moyens de prévision de la couche limite turbulent tridimensionelle. D.Sc. thesis, University of Paris.

COUSTEIX, J. & MICHEL, R.* Theoretical analysis and prediction of three-dimensional turbulent boundary layers.

EAST, L. F.* Progress report on the double split-film probe.

EAST, L. F.* 1975 Computation of three-dimensional turbulent boundary layers. *Euromech* 60, *Trondheim.* To be published.

ELSENAAR, A. & BOELSMA, S. H.* 1974 Measurements of the Reynolds stress tensor in a three-dimensional turbulent boundary layer under infinite swept wing conditions. *N.L.R. Amst. Tech. Rep.* no. 74095U.

FANNELØP, T. K. & HUMPHREYS, D. 1975 The solution of the laminar and turbulent three-dimensional boundary layer equations with a simple finite difference technique. *F.F.A., Aero. Res. Inst. Sweden, Stockholm, Rep.* no. 126.

FERNHOLZ, H.* From Euromech 33 to Euromech 60. A personal view of progress (or lack of it) in three-dimensional boundary-layer research.

GRANDE, G. DE* Three-dimensional turbulent boundary-layer decay.

HESS, J. L. & SMITH, A. M. O. 1967 Calculation of potential flow about arbitrary bodies. *Prog. Aero. Sci.* **8**, 1–138.

HIRSCHEL, E. H.* Transition criteria for three-dimensional boundary layers.

HUMPHREYS, D.* Approximations in three-dimensional turbulent boundary layer calculations.

KRAUSE, E. & KORDULLA, W.* Investigation of three-dimensional boundary layers on swept wings.

KROGSTAD, P. Å.* Three-dimensional viscous flow fields driven by simple two-dimensional potential flows.

KUX, J. & HOFFMANN, H. P.* The boundary layer on a model ship hull.

LARSSON, L. 1974 Boundary layers of ships. Part III. An experimental investigation of the turbulent boundary layer on a ship model. *Rep. Swedish State Ship Building Exp. Tank, Gothenburg.*

LARSSON, L.* Measurements and calculations of the three-dimensional turbulent boundary layer on a ship model.

LAUNDER, B. E. & SPALDING, D. B. 1972 *Mathematical Models of Turbulence.* Academic.

MICHEL, R., COUSTEIX, J. & QUÉMARD, C. 1971 Application d'un schéma amélioré de longueur de mélange à l'étude des couches limites turbulentes tridimensionelles. *Specialists' Meeting on Turbulent Shear Flows, London*, paper 7 (*AGARD Conf. Proc.* no. 93).

MYRING, D. B. 1970 An integral prediction method for three-dimensional turbulent boundary layers in incompressible flow. *R.A.E. Tech. Rep.* no. 70/47.

PALM, E.* On the breakdown of streamwise vortices.

PATANKAR, S. V. & SPALDING, D. B. 1972 A calculation procedure for heat, mass and momentum transfer in three-dimensional parabolic flows. *J. Heat Mass Transfer*, **15**, 1787–1806.

PERSEN, L. N. 1970 Streamwise directed vortices and crosshatched surfaces of reentry vehicles. *J. Spacecraft Rockets*, **7**, 108–110.

PERSEN, L. N. 1971 Surface patterns of ablating bodies studied by means of water experiment simulation. *Z. Flugwiss.* **19**, 360–373.

PERSEN, L. N.* Can we learn from streamwise directed vortices?

RODI, W., RASTOGI, A. K., SINGHAL, A. K. & SPALDING, D. B.* Application of the k, ϵ turbulence model to the prediction of three-dimensional external boundary layers.

SCHMITT, V. & COUSTEIX, J.* Experimental and theoretical study of three-dimensional phenomena related to the development of boundary layers on a swept wing up to high angles of attack.

SCHNEIDER, G.* A finite-difference method with a mixing-length hypothesis for the calculation of three-dimensional incompressible turbulent boundary layers.

SMITH, P. D. 1974 An integral prediction method for three-dimensional compressible turbulent boundary layers. *Aero. Res. Counc. R. & M.* no. 3739.

SMITH, P. D.* Calculation methods for three-dimensional turbulent boundary layers.

SPALDING, D. B.* Calculation procedure for three-dimensional parabolic and partially parabolic flows.

THOMANN, H.* Three-dimensional effects in a 'two-dimensional' boundary layer.

VERMEULEN, A. J. 1971 Measurements of three-dimensional turbulent boundary layers. Ph.D. thesis, University of Cambridge.

WHITE, F. M. & CHRISTOPH, G. H. 1971 A simple analysis of two-dimensional turbulent skin friction with arbitrary wall and freestream conditions. *Specialists' Meeting on Turbulent Shear Flows, London*, paper 20 (*AGARD Conf. Proc.* no. 93).

WHITE, F. M., CHRISTOPH, G. H. & LESSMANN, R. C.* 1975 A three-dimensional integral method for calculating incompressible turbulent skin friction. *A.S.M.E.* no. 75-FE-26.